

Quark-gluon mixed condensate of the QCD vacuum in Holographic QCD

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ABSTRACT: We investigate the quark-gluon mixed condensate based on an AdS/QCD model. Introducing a holographic field dual to the operator for the quark-gluon mixed condensate, we obtain the corresponding classical equation of motion. Taking the mixed condensate as an additional free parameter, we show that the present scheme reproduces very well experimental data. A fixed value of the mixed condensate is in good agreement with that of the QCD sum rules.

KEYWORDS: AdS-CFT Correspondence, Quark-gluon mixed condensates, Nonperturbative QCD.

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1. Introduction

The QCD vacuum is known to be very complicated due to both perturbative and non-perturbative fluctuations. In particular, the quark and gluon condensates, being the lowest dimensional ones, feature the non-perturbative structure of the QCD vacuum. The quark condensate can be identified as the order parameter for the spontaneous breakdown of chiral symmetry (SB χ S) which plays an essential role in describing low-energy phenomena of hadrons: In the QCD sum rule, the chiral condensate arises from the operator product expansion and is determined phenomenologically by hadronic observables [1, 2], while in chiral perturbation theory (χ PT), it is introduced in the mass term of the effective chiral Lagrangian at the leading order [3, 4].

While the quark and gluon condensates are well understood phenomenologically, higher dimensional condensates suffer from large uncertainty. Though it is possible to estimate dimension-six four-quark condensates in terms of the quark condensate by using the factorization scheme that is justified in the large N_c limit, the dimension-five mixed quark-gluon condensate is not easily determined phenomenologically [5, 6, 9, 10, 11, 12, 13]. In particular, the mixed condensate is an essential parameter to calculate baryon masses [5], exotic hybrid mesons [7], higher-twist meson distribution amplitudes [8] within the QCD sum rules. Moreover, the mixed quark condensate can be regarded as an additional order parameter for the SB χ S since the quark chirality flips via the quark-gluon operator. Thus, it is naturally expressed in terms of the quark condensate:

$$\langle \bar{\psi} \sigma^{\mu\nu} G_{\mu\nu} \psi \rangle = m_0^2 \langle \bar{\psi} \psi \rangle \quad (1.1)$$

with the dimensional parameter m_0^2 which was estimated in various works [5, 6, 9, 10, 11, 12, 13]. The gluon-field strength is defined as $G_{\mu\nu} = G_{\mu\nu}^a \lambda^a / 2$.

The AdS/CFT correspondence [14, 15, 16] that provides a connection between a strongly coupled large N_c gauge theory and a weakly coupled supergravity gives new and attractive insight into nonperturbative features of quantum chromodynamics (QCD) such as the quark confinement and spontaneous breakdown of chiral symmetry (SB χ S). Though

there is still no rigorous theoretical ground for such a correspondence in real QCD, this new idea has triggered a great amount of theoretical works on possible mappings from nonperturbative QCD to 5D gravity, i.e. holographic dual of QCD. In fact, there are in general two different routes to modeling holographic dual of QCD (See, for example, a recent review [17]): One way is to construct 10 dimensional (10D) models based on string theory of D3/D7, D4/D6 or D4/D8 branes [18, 19, 20, 21, 22]. The other way is so-called a bottom-up approach to a holographic model of QCD, often called as AdS (Anti-de Sitter Space)/QCD [23, 24, 25] in which a 5D holographic dual is constructed from QCD, the 5D gauge coupling being identified by matching the two-point vector correlation functions. Despite the fact that this bottom-up approach is somewhat on an ad hoc basis, it reflects some of most important features of gauge/gravity dual. Moreover, it is rather successful in describing properties of hadrons (See the recent review [17]).

In the present work, we aim at investigating the mixed condensate of the QCD vacuum defined as Eq.(1.1), closely following ref. [23]. The hard-wall model of ref. [23] is the simplest one but provides a clean-cut framework to study the mixed condensate. Thus, we will extend the 5D action in ref. [23], introducing the bulk field corresponding to the operator for the mixed condensate. We will carefully examine how the meson masses and couplings undergo changes in the presence of the mixed condensate.

We sketch the present work as follows: In section 2, we describe briefly the hard-wall AdS/QCD model with the bulk field for the mixed condensate taken into account. In section 3, we show our results for the mixed condensate. we also examine, in the presence of the mixed condensate, possible changes of the meson observables such as masses and coupling constants. In the last section, we summarize the present work and draw conclusions.

2. A hard-wall AdS/QCD model

The metric of an AdS space is given as

$$ds^2 = g_{MN}dx^M dx^N = \frac{1}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2), \quad (2.1)$$

where $\eta_{\mu\nu}$ denotes the 4D Minkowski metric: $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. The AdS space is compactified by two different boundary conditions, i.e. the IR boundary at $z = z_m$, and the UV at $z = \epsilon \rightarrow 0$. Thus, the model is defined within the range: $\epsilon \leq z \leq z_m$. Taking into account the bulk field corresponding to the operator for the mixed condensate, we express the classical 5D bulk action as follows:

$$S = \int d^5x \sqrt{g} \text{Tr} \left[|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) + |D\Phi|^2 - 5\Phi^2 \right], \quad (2.2)$$

where $D_\mu X = \partial_\mu X - iA_{L\mu}X + iXA_{R\mu}$ and $F_{L,R}^{\mu\nu} = \partial^\mu A_{L,R}^\nu - \partial^\nu A_{L,R}^\mu - i[A_{L,R}^\mu, A_{L,R}^\nu]$. The massless gauge field is defined as $A_{L,R} = A_{L,R}^a t^a$ with $\text{tr}(t^a t^b) = \delta^{ab}/2$. The 5D masses m_5^2 of the bulk fields are determined by the relation $m_5^2 = (\Delta - p)(\Delta + p - 4)$ [15, 16], where Δ stands for the dimension of the corresponding operator with spin p . In table 1, the

4D operators: $\mathcal{O}(x)$	5D fields: $\phi(x, z)$	p	Δ	m_5^2
$\bar{q}_L \gamma^\mu t^a q_L$	$A_{L\mu}^a$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	$A_{R\mu}^a$	1	3	0
$\bar{q}_R^\alpha q_L^\beta$	$(2/z) X^{\alpha\beta}$	0	3	-3
$\bar{q}_R^\alpha \sigma_{\mu\nu} G^{\mu\nu} q_L^\beta$	$(1/z^3) \Phi^{\alpha\beta}$	0	5	5

Table 1: A dictionary for 4D operators and 5D fields

operators and corresponding bulk fields are listed with the 5D masses given. g_5 represents the 5D gauge coupling. The bi-fundamental scalar field X is relevant to SB χ S. Its vacuum expectation value (VEV) is holographic dual to the bilinear scalar quark operator $\bar{q}_L q_R$, which can be written in terms of the chiral condensate $\langle \bar{q}_L q_R \rangle = \Sigma = \sigma \mathbf{1}$ and the current quark mass $\hat{m} = \text{diag}(m_q, m_q)$. The bi-fundamental scalar field $\Phi(x, z)$ is introduced as a holographic dual for the operator of the mixed quark-gluon condensate $\bar{q}_R \sigma_{\mu\nu} G^{\mu\nu} q_L$. In this work, we focus on the VEV of $\Phi(x, z)$ to study the mixed condensate.

In ref. [26], it was shown that for small z or near the boundary of AdS space, a 5D field $\phi(x, z)$ dual to a 4D operator \mathcal{O} can be expressed as

$$\phi(x, z) = z^{4-\Delta} [\phi_0(x) + O(z^2)] + z^\Delta [A(x) + O(z^2)], \quad (2.3)$$

where $\phi_0(x)$ is a prescribed source function for $\mathcal{O}(x)$ and $A(x)$ denotes a physical fluctuation that can be determined from the source by solving the classical equation of motion. It can be directly related to the VEV of the $\mathcal{O}(x)$ as follows [26]:

$$A(x) = \frac{1}{2\Delta - 4} \langle \mathcal{O}(x) \rangle. \quad (2.4)$$

Thus, from the classical equations of motion of X [23, 24] and Φ , we obtain

$$\begin{aligned} X_0(x, z) &= \langle X(x, z) \rangle = \frac{1}{2}(\hat{m}z + \sigma z^3), \\ \Phi_0(x, z) &= \langle \Phi(x, z) \rangle = \frac{1}{6}(c_1 z^{-1} + \sigma_M z^5), \end{aligned} \quad (2.5)$$

where c_1 is the source term for the mixed condensate and σ_M represents the mixed condensate $\sigma_M = \langle \bar{q}_R \sigma_{\mu\nu} G^{\mu\nu} q_L \rangle$. For simplicity, we take $c_1 = 0$.

We now review how to fix the 5D gauge coupling [23, 24] by matching the two-point vector correlation function to the leading contribution from the OPE result [1]. The vector field V is defined as $(A_L + A_R)/2$ with the axial-like gauge condition $V_z(x, z) = 0$. It can be decomposed into the transverse and longitudinal parts: $V_\mu = (V_\mu)_\perp + (V_\mu)_\parallel$. Using the Fourier transform of the vector field: $V_\mu^a = \int d^4x e^{iq \cdot x} V_\mu^a(x, z)$, we can write the equation of motion for the transverse part of the vector field as follows [23]:

$$\left[\partial_z \left(\frac{1}{z} \partial_z V_\mu^a(q, z) \right) + \frac{q^2}{z} V_\mu^a(q, z) \right]_\perp = 0. \quad (2.6)$$

The corresponding solution of Eq.(2.6) can be expressed as a separable form:

$$(V_\mu^a(q, z))_\perp = V(q, z) \bar{V}_\mu^a(q), \quad (2.7)$$

where $\bar{V}_\mu^a(q)$ is the Fourier transform of the source of the 4D conserved vector current operator $\bar{q}\gamma_\mu t^a q$ at the boundary. The explicit solution for $V(q, z)$ can be derived by solving Eq.(2.6) with the boundary conditions $V(q, \epsilon) = 1$ and $\partial_z V(q, z_m) = 0$:

$$V(q, z) = \frac{\pi q z}{2} \left[\frac{Y_0(q z_m)}{2J_0(q z_m)} J_1(q z) - Y_1(q z) \right], \quad (2.8)$$

where J_i and Y_i denote the Bessel functions of the first and second kinds, respectively. The $V(q, z)$ is often called a bulk-to-boundary propagator, since the solution $V(q, z)$ leaves only the boundary term of the action in a quadratic form:

$$S_b = -\frac{1}{2g_5^2} \int d^4 x \bar{V}_\mu(q) \left(\frac{\partial_z V(q, z)}{z} \right)_{z=\epsilon} \bar{V}^\mu(q). \quad (2.9)$$

Thus, the correlation function can be obtained by the second derivative of the action with respect to the vector field $\bar{V}^\mu(q)$:

$$\Pi_V(Q^2) = -\frac{1}{g_5^2 Q^2} \frac{\partial_z V(q, z)}{z}, \quad (2.10)$$

where $Q^2 = -q^2$. In a large Euclidean region ($Q^2 \rightarrow \infty$), one gets

$$\Pi_V(Q^2) = -\frac{1}{2g_5^2} \ln Q^2. \quad (2.11)$$

Since it is already known from the OPE that the vector correlation function in the leading order is given as

$$\Pi_V(Q^2) = -\frac{N_c}{24\pi^2} \ln Q^2, \quad (2.12)$$

one can immediately determine the 5D gauge coupling g_5^2 :

$$g_5^2 = \frac{12\pi^2}{N_c}. \quad (2.13)$$

We are now in a position to derive the classical equations of motion for the axial-vector and pion. Introducing $v = m_q z + \sigma z^3$, $w = (\sigma_M/3)z^5$, and $(A_\mu)_\parallel = \partial_\mu \phi$, we obtain

$$\left[\partial_z \left(\frac{1}{z} \partial_z A_\mu \right) + \frac{q^2}{z} A_\mu - g_5^2 \frac{1}{z^3} (v^2 + w^2) A_\mu \right]_\perp = 0, \quad (2.14)$$

$$\begin{aligned} \partial_z \left(\frac{1}{z} \partial_z \phi^a \right) + g_5^2 \frac{1}{z^3} v^2 (\pi^a - \phi^a) &= 0, \\ -q^2 \partial_z \phi^a + g_5^2 \frac{1}{z^2} (v^2 + w^2) \partial_z \pi^a &= 0. \end{aligned} \quad (2.15)$$

Finally, we consider decay constants and interactions in the model [23, 24]. The decay constant of ρ is given by

$$F_\rho^2 = \frac{1}{g_5^2} \left(\frac{\psi'_\rho(\epsilon)}{\epsilon} \right)^2, \quad (2.16)$$

where $\psi_\rho(z)$ denotes a ρ -meson wave function defined as: $V_\mu(x, z) = g_5 \sum_n V_\mu^{(n)}(x) \psi^{(n)}(z)$. The ρ -meson wave function is the solution of (2.6) at $q^2 = m_\rho^2$ with the boundary conditions $\psi_\rho(\epsilon) = 0$ and $\partial_z \psi_\rho(z_m) = 0$ imposed. The pion decay constant is

$$f_\pi^2 = -\frac{1}{g_5^2} \frac{\partial_z A(0, z)}{z} \Big|_{z=\epsilon}, \quad (2.17)$$

where $A(0, z)$ is the solution of (2.14) with $q^2 = 0$ and with two boundary conditions: $A(0, \epsilon) = 1$, $A'(0, z_m) = 0$. The $\pi - \rho$ coupling reads as follows [23, 24]:

$$g_{\rho\pi\pi} = g_5 \int_\epsilon^{z_m} dz \psi_\rho(z) \left(\frac{\phi'^2}{g_5^2 z} + \frac{v(z)^2 (\pi - \phi)^2}{z^3} \right). \quad (2.18)$$

3. Numerical Results

In this section, we present the numerical results of various hadronic observables and condensates discussed previously.

	Model I	Model II	Model III	Model IV	Experiment
m_q	1.6	3.7	2.3	2.3	...
σ	$(0.1 \text{ GeV})^3$	$(0.25 \text{ GeV})^3$	$(0.307 \text{ GeV})^3$	$(0.308 \text{ GeV})^3$...
m_0^2	13.32 GeV^2	0.72 GeV^2	0.006 GeV^2	0	...
m_ρ	775.8	775.8	775.8	832	775.49 ± 0.34
m_{a_1}	1230	1244	1246	1220	1230 ± 40
f_π	75.9	80.5	85.5	84.0	92.4 ± 0.35
$F_\rho^{1/2}$	330	330	330	353	345 ± 8
$F_{a_1}^{1/2}$	460	459	446	440	433 ± 13
m_π	138	139.3	137.5	141	139.57 ± 0.00035
$g_{\rho\pi\pi}$	8.27	4.87	4.87	5.29	6.03 ± 0.07
g_{A4}	1.71	1.69	1.71	1.88	...

Table 2: The results of the model with and without the mixed condensate. Model IV corresponds to Model B of ref. [23]. The experimental data listed in the last column are taken from the particle data group [27]. The empirical decay constants and coupling constants are extracted from the corresponding decay widths [23]. All results are given in units of MeV except for the condensate and the ratio of two condensates.

Our results are summarized in table 2. We use the ρ -meson mass as an input and do the global fit to the other observables as in Ref. [23]. We obtain three different sets of the results for which we call Model I, II, and III. For the sake of comparison, we also list the results of ref. [23] as Model IV. Note that the value of the ratio between the quark condensate and mixed quark-gluon condensate, *i.e.* m_0^2 , has not been uniquely determined [5, 6, 9, 10, 11, 12, 13]. For example, while Belyaev and Ioffe have predicted $m_0^2 \simeq 0.8 \text{ GeV}^2$, based on the QCD sum rules [6], Doi et al. have obtained $m_0^2 \simeq 2.5 \text{ GeV}^2$ [12] in quenched lattice QCD.

In the present study, it turns out that the value of m_0^2 is mostly fixed by the mass of a_1 . The pion decay constant from Model I seems to be quite underestimated in comparison with the data. Moreover, the corresponding result of m_0^2 becomes much larger than those of other works. Thus, it implies that Model I seems to be ruled out. On the other hand, the results from Model II and Model III are in qualitative agreement with measured observables. However, the values of m_0^2 are rather different each other.

Since the ratio m_0^2 is sensitive to a_1 mass, we calculate a coupling involving a_1 to see if our model can select a particular value of σ_M/σ . The coupling of four a_1 fields can be determined in the following way:

$$\begin{aligned} S_{A4}^{4D} &= g_{A4} \text{Tr} \int d^4x A(x)_\mu A(x)_\nu A(x)_\mu A(x)_\nu, \\ g_{A4} &\equiv 2 \int_\epsilon^{z_m} dz \frac{1}{z} \psi_{a_1}(z)^4. \end{aligned} \quad (3.1)$$

Since we have already calculated the a_1 meson wave function, it is straightforward to read out the results for g_{A4} . The values of g_{A4} are listed in the last row of table 2. As shown in the table, the value of g_{A4} is almost stable with m_0^2 varied. However, since the m_0^2 from Model II is comparable to that of the QCD sum rules [6], Model II is favored in the present work.

4. Summary and Conclusion

In the present work, we have aimed at studying the quark-gluon mixed condensate within the framework of the hard-wall AdS/QCD model [23, 24]. To this end, we have introduced a bulk scalar field Φ dual to the mixed condensate to the hard wall model. We have treated the ratio of the chiral and mixed condensates m_0^2 as a free parameter and fixed it by phenomenology. Our results are summarized in table 2. Model II in the table predicts $m_0^2 \simeq 0.72 \text{ GeV}^2$, which is comparable to that from the QCD sum rules $m_0^2 \simeq 0.8 \text{ GeV}^2$ [6]. It indicates that Model II is the most favorable one, though we were not able to uniquely fix the value of the mixed condensate in the present work. However, the present simple hard-wall model seems to be not adequate to fix the mixed condensate uniquely. Nevertheless, we conclude that the mixed condensate should be considered as an important ingredient of low-energy QCD as well as the chiral condensate in any AdS/QCD models.

Finally, we remark that it will be interesting to see if one can study the mixed condensate in a stringy set-up. This is due to the fact that, unlike the chiral or gluon condensate, the mixed quark-gluon condensate is associated with two completely different branes such as $D3 - D7$ or $D4 - D8$.

Acknowledgments

The present work is supported by Inha University Research Grant (INHA-37453).

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